Bias on the Wind-Speed Gust caused by Quantization and Disjunct Sampling of the Cup-Anemometer Signal

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Presentation of Main Results

We are here concerned with large excursions of the wind speed as measured by a cup anemometer. This instrument, which in this context may be assumed to have a linear calibration with zero offset, detects the wind speed through its first-order filter with a time constant given by*

$$\tau_\circ = \frac{\ell_\circ}{U}, \tag{1}$$

where $\ell_\circ$, the distance constant, is an instrument constant and U the mean-wind speed.

Large excursions are usually characterized by the so-called gust, and to define this quantity we need to consider the time series $u(t)$ of the wind speed over a finite time T. In this situation we want to determine how often a certain level $\mathcal{U}$ is exceeded. We consider an (infinite) ensemble of stationary time series where one member is shown Fig. 1.

Imagining that we have an infinite ensemble of realizations of $u(t)$ with the mean U, observed over the time T, we can determine the average number of times the level $\mathcal{U}$ is exceeded by

$$N(\mathcal{U}) = \lim_{M\to\infty} \frac{1}{M}\sum_{i=1}^{M} N_i(\mathcal{U}), \tag{2}$$

where $N_i(\mathcal{U})$ is the number of times $\mathcal{U}$ is exceeded in the i'th realization.

We use the definition:

The gust $[\mathcal{U}]$ is the wind speed deviation from the mean which—on average—is exceeded once during the reference period T.

The cup anemometer is characterized by a linear calibration between the rate of rotation and the wind speed. Typically this rate is determined by a rate f of pulses, generated by the rotor, with one pulse for each time it has turned a certain fraction of a full rotation. For a constant wind speed U the calibration equation is a linear relation between U and the frequency f:

*The offset is seldom more than about 0.2 m s^{-1} and is not of importance in these considerations. The effect of the offset on the time constant is briefly discussed by Kristensen (1998).

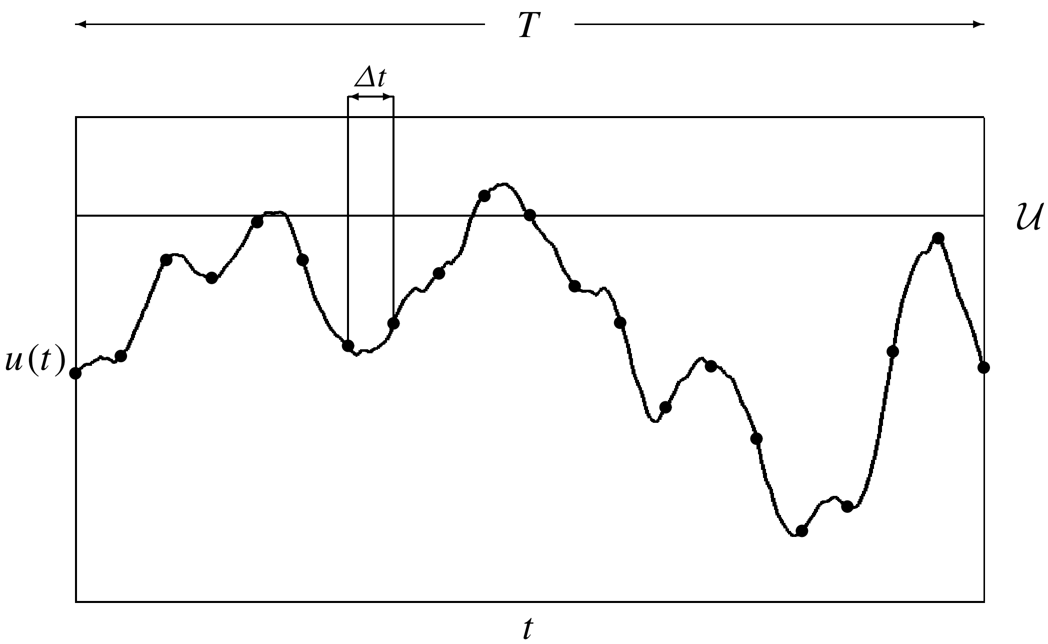

Figure 1: Sample time series of the wind speed $u(t)$ as recorded over the period T. The wind-speed level $\mathcal{U}$ is indicated. The wind speed is recorded at times separated by the time interval Δt. These are indicated by dots. This example shows that the continuous signal exceeds the level $\mathcal{U}$ twice, but that the discrete signal only detects one excursion beyond $\mathcal{U}$,

$$U = B \times f, \tag{3}$$

where B is a constant length.

When sampling the cup-anemometer signal it is averaged over the time interval Δt and recorded by the end of the averaging interval. Mathematically this corresponds to letting the signal undergo a running-mean process of duration Δt, followed by instantaneous read-outs separated by Δt. Signal recordings separated by a fixed time interval is called *disjunct sampling* (Lenschow et al. 1994)[†]. The raw wind-speed is consequently subjected to two different low-pass filters, this running-mean process and the preceding first-order filtering with the time constant $\tau_\circ$, given by (1). Low-pass filtering will in itself influence the gust magnitude: larger $\tau_\circ$ and/or Δt imply a smaller gust determination (Kristensen et al. 1991). This gust-suppressing filtering effect is in principle unavoidable. In the following we discuss gust bias of this doubly filtered signal.

There are two distinct reasons for a bias on the gust value, namely

1. the quantization of the cup anemometer signal, and
2. the disjunct sampling.

[†]It has been suggested to call this type of recording 'intermittent sampling'. However, since the word 'intermittent' suggests a certain amount of randomness, we prefer the adjective 'disjunct' to emphasize that we are dealing with "contiguous parts marked by separation" (Woolf 1977).

The first bias source is related to the calibration (3). Since the resolution in the frequency f is equal to $1/\Delta t$, the corresponding resolution in U is

$$\Delta = \frac{B}{\Delta t}. \tag{4}$$

The quantization gives rise to an additional variance given by the quantum Δ. It is well-known that this amounts to $\Delta^2/12$ (Sheppard 1898). The corresponding extra fluctuation will in principle enhance statistically the maximum measured value.

The second source is caused by the fact that by disjunct sampling there is a high probability of missing the maximum value because in all likelihood it will fall between recordings, separated in time by Δt.

We see that the quantization gives rise to a positive gust bias whereas the opposite is the case for the disjunct sampling. The biases are determined in the framework of the classical theory by S.O. Rice (Rice 1944, 1945) for the rate of excursions of a continuous $u(t)$ beyond the level $\mathcal{U}$ by

$$\eta(\mathcal{U}) = \frac{N(\mathcal{U})}{T}. \tag{5}$$

It is convenient at this point to define the dimensionless gust, the so-called gust factor, by

$$\mu = \frac{[\mathcal{U}] - U}{\sigma_u} \tag{6}$$

as the deviation from the mean U, normalized by the root-mean-square σ_u.

The following sections provide technical documentation and confirm that the gust bias from quantization is positive and that the disjunct sampling will lead to an underestimation of the gust. Figs. 2 and 4 summarize the results for the Risø model P2546 with the distance constant $\ell_\circ = 1.8$ m and the calibration constant $B = 0.62$ m. We have found that the signal quantization will overestimate the gust with less than 1% when the wind speed U is larger than 2 m s^{-1} and the time between samples Δt is larger than 1 s. This bias is a decreasing function of U and Δt as shown in Fig. 2. The bias due to disjunct sampling is negative and its absolute value an increasing function of Δt. According to Fig. 4 it is less than 5% in cases of practical importance.

The Rice Approach

The Rice approach is textbook literature and it is beyond the scope here to present a complete list of relevant references. A concise derivation of an equation has been given by Panofsky & Dutton (1984). Here is a short version of the theory: Let the joint probability density for $u(t)$ and its derivative $\dot{u}(t)$ be $p(u, \dot{u})$. Then the probability that $u(t)$ performs one crossing of the level $\mathcal{U}$ in the short time interval from $t - \Delta t$ to t with a time derivative in the interval from $\dot{u}$ to $\dot{u} + \mathrm{d}\dot{u}$ is

$$\mathrm{d}\dot{u} \int\limits_{\mathcal{U}-\dot{u}\Delta t}^{\mathcal{U}} p(u,\dot{u})\,\mathrm{d}u. \tag{7}$$

Consequently, the probability that $u(t)$ performs one up-crossing ($\dot{u} > 0$) of $\mathcal{U}$ in the time interval Δt becomes

$$\int\limits_0^\infty \mathrm{d}\dot{u} \int\limits_{\mathcal{U}-\dot{u}\Delta t}^{\mathcal{U}} p(u,\dot{u})\,\mathrm{d}u \simeq \Delta t \int\limits_0^\infty \dot{u}\; p(\mathcal{U},\dot{u})\,\mathrm{d}\dot{u}. \tag{8}$$

This leads directly to an equation for average rate of excursions beyond $\mathcal{U}$:

$$\eta(\mathcal{U}) = \int\limits_0^\infty \dot{u}\; p(\mathcal{U},\dot{u})\,\mathrm{d}\dot{u}. \tag{9}$$

In this section it has been implicitly assumed that $u(t)$ and $\dot{u}(t)$ are continuous and sampled continuously in time.

To proceed we need to know the joint probability-density function $p(u,\dot{u})$. Here we just assume that it is $p(u,\dot{u})$ is Gaussian in both u and $\dot{u}$. For a stationary time series $u(t)$ all mean values are constant in time. In particular, the signal $u(t)$ and the squared signal $u(t)^2$ have constant mean values $\langle u\rangle = U$ and $\langle u^2\rangle$, respectively. This means that u and $\dot{u}$ are uncorrelated since

$$0 = \frac{\mathrm{d}}{\mathrm{d}t}\langle u^2\rangle = 2\langle u\dot{u}\rangle = 2\langle (u-U)\dot{u}\rangle, \tag{10}$$

where we have used the fact that the mean of $\dot{u}$ is zero since it is a time derivative of a constant. With the assumption that $p(u,\dot{u})$ is Gaussian we have

$$p(u,\dot{u}) = \frac{1}{2\pi}\,\frac{1}{\sigma_u\,\langle \dot{u}^2\rangle^{1/2}}\exp\left(-\frac{(u-U)^2}{2\sigma_u^2} - \frac{\dot{u}^2}{2\langle \dot{u}^2\rangle}\right), \tag{11}$$

where $\sigma_u^2 = \langle (u-U)^2\rangle = \langle u^2\rangle - U^2$ is the variance of $u(t)$. Substituting (11) into (9), we obtain

$$\eta(\mathcal{U}) = \frac{1}{2\pi}\frac{\langle \dot{u}^2\rangle^{1/2}}{\sigma_u}\exp\left(-\frac{(\mathcal{U}-U)^2}{2\sigma_u^2}\right). \tag{12}$$

According to the definition of the gust $[\mathcal{U}]$, this quantity is obtained from (12) by the identity

$$1 \equiv \eta([\mathcal{U}]) \times T = \frac{1}{2\pi} \frac{\langle \dot{u}^2 \rangle^{1/2} T}{\sigma_u} \exp\left(- \frac{([\mathcal{U}] - U)^2}{2\sigma_u^2} \right). \tag{13}$$

The gust factor $\mu = ([\mathcal{U}] - U)/\sigma_u$ is given by

$$\mu^2 = 2 \ln\left(\frac{1}{2\pi} \frac{\langle \dot{u}^2 \rangle^{1/2} T}{\sigma_u} \right). \tag{14}$$

Now we are only left with the task of determining $\langle \dot{u}^2 \rangle$, the variance of the time derivative of $u(t)$. This will require knowledge of the power spectrum $S(\omega)$ of $u(t)$. Let the true, one-sided power spectrum of the wind speed be $S_\circ(\omega)$. Then the power spectrum of the doubly filtered signal is

$$S(\omega) = \frac{S_\circ(\omega)}{1 + (\ell_\circ\, \omega/U)^2} \frac{\sin^2(\omega \Delta t/2)}{(\omega \Delta t/2)^2}. \tag{15}$$

We can determine $\langle \dot{u}^2 \rangle$ by means of the expression

$$\begin{aligned}
\langle \dot{u}^2 \rangle &= \int_0^\infty \omega^2\, S(\omega)\, \mathrm{d}\omega \\
&= \int_0^\infty \frac{\omega^2 S_\circ(\omega)}{1 + (\ell_\circ\, \omega/U)^2} \frac{\sin^2(\omega \Delta t/2)}{(\omega \Delta t/2)^2}\, \mathrm{d}\omega \\
&= \frac{2}{\Delta t^2} \int_0^\infty \frac{S_\circ(\omega)}{1 + (\ell_\circ\, \omega/U)^2} [1 - \cos(\omega \Delta t)]\, \mathrm{d}\omega.
\end{aligned} \tag{16}$$

To evaluate (16) we need to specify the unfiltered power spectrum only at high frequencies and we use the standard form for the so-called inertial subrange

$$S_\circ(\omega) = \alpha_1\, \varepsilon^{2/3} \left(\frac{\omega}{U} \right)^{-5/3} \frac{1}{U}, \tag{17}$$

where ε is the rate of dissipation of specific kinetic energy, or just dissipation for short, and $\alpha_1 \simeq 0.56$ the dimensionless Kolmogorov constant for the one-dimensional, one-sided spectrum of the flow-wise velocity component. Inserting (17) in (16), we get

$$\langle \dot{u}^2 \rangle \Delta t^2 = 2\alpha_1 \, (\varepsilon \ell_\circ)^{2/3} \mathcal{I}\!\left(\frac{U \Delta t}{\ell_\circ}\right), \tag{18}$$

where

$$\begin{aligned} \mathcal{I}(q) &= \int_0^\infty \frac{1-\cos(qs)}{1+s^2}\, s^{-5/3}\, \mathrm{d}s \\ &= \frac{\pi}{\sqrt{3}}\,(\cosh(q)-1) - \frac{27}{160}\,\Gamma\!\left(\frac{1}{3}\right) q^{8/3}\, {}_1F_2\!\left(1; \frac{11}{6}, \frac{7}{3}; \frac{q^2}{4}\right). \end{aligned} \tag{19}$$

Here ${}_1F_2(a; b, c; x)$ is a generalized hypergeometric function.

For wind speeds larger than a few meters per second the atmosphere may be assumed neutrally stratified, and in this case ε may be expressed in terms of the height z over the ground of the anemometer and the friction velocity u_*, which incidentally is also related to the mean-wind speed

$$U(z) = \frac{u_*}{\kappa} \ln\!\left(\frac{z}{z_\circ}\right) \tag{20}$$

over a terrain with the roughness length $z_\circ$. The dimensionless constant $\kappa \simeq 0.4$ is the so-called von Kármán constant. In this case the dissipation is given by (Panofsky & Dutton 1984)

$$\varepsilon = \frac{u_*^3}{\kappa z}. \tag{21}$$

With (21) we may rewrite (18) as

$$\langle \dot{u}^2 \rangle \Delta t^2 = \underbrace{2\frac{\alpha_1}{\kappa^{2/3}}}_{\simeq 2.1} u_*^2 \left(\frac{\ell_\circ}{z}\right)^{2/3} \mathcal{I}\!\left(\frac{U \Delta t}{\ell_\circ}\right). \tag{22}$$

According to Panofsky & Dutton (1984)

$$\sigma_u \simeq 2.4\, u_* \tag{23}$$

and with this additional information (14) can easily be evaluated:

$$\begin{aligned}\mu^2 &= \ln\left(\frac{1}{(2\pi)^2}\frac{2\alpha_1}{\kappa^{2/3}}\frac{1}{2.4^2}\left(\frac{T}{\Delta t}\right)^2\left(\frac{\ell_\circ}{z}\right)^{2/3}\mathcal{I}\left(\frac{U\Delta t}{\ell_\circ}\right)\right)\\ &= C+\frac{2}{3}\ln\left(\frac{\ell_\circ}{z}\right)+2\ln\left(\frac{UT}{\ell_\circ}\right)+\ln\left(\frac{\mathcal{I}\left(\frac{U\Delta t}{\ell_\circ}\right)}{\left(\frac{U\Delta t}{\ell_\circ}\right)^2}\right),\end{aligned} \tag{24}$$

where

$$C=\ln\left(\frac{1}{(2\pi)^2}\frac{2\alpha_1}{\kappa^{2/3}}\frac{1}{2.4^2}\right)\simeq -4.7 \tag{25}$$

is a dimensionless constant.

Quantization Bias

As pointed out in the introduction, the quantization of the signal adds to the measured variance with an amount equal to $\Delta^2/12$. This information, however, is in itself not sufficient to determine the quantization contribution to the gust. We must also know the frequency distribution of this variance, i.e. its power spectrum. Kristensen & Kirkegaard (1987) found that for a digitized signal one can account for the extra spectral contribution by adding the auto-covariance

$$\delta R_\circ(\tau)\simeq\begin{cases}\frac{\Delta^2}{12}\left(1-\frac{|\tau|}{\tau_*}\right), & |\tau|\le\tau_*\\ 0, & |\tau|>\tau_*\end{cases}. \tag{26}$$

to the auto-covariance of $u(t)$. Here the time scale τ_* is given by

$$\tau_*=\lambda\,\frac{\Delta}{\sigma_u}, \tag{27}$$

where λ is the Taylor microscale, which is given by the relation (Tennekes & Lumley 1978)

$$\frac{\sigma_u^2}{\lambda^2}=\frac{\langle\dot{u}^2\rangle}{2}. \tag{28}$$

The extra auto-covariance (26) corresponds to the additional spectral contribution

$$\delta S_\circ(\omega) = \frac{\Delta^2}{12}\,\frac{\tau_*}{2\pi}\,\frac{\sin^2\!\left(\frac{\omega\tau_*}{2}\right)}{\left(\frac{\omega\tau_*}{2}\right)^2}. \tag{29}$$

It follows, by replacing $S_\circ(\omega)$ by $\delta S_\circ(\omega)$ in (16), that an extra variance $\delta\langle\dot{u}^2\rangle$ must be included in the variance of $\dot{u}$. The extra variance is then given by

$$\begin{aligned}
\delta\langle\dot{u}^2\rangle\Delta t^2 &= 2\int_0^\infty \frac{\delta S_\circ(\omega)}{1+(\ell_\circ\,\omega/U)^2}\,[1-\cos(\omega\Delta t)]\,\mathrm{d}\omega \\
&= \frac{\Delta^2}{6\pi}\int_0^\infty \frac{\{1-\cos s\}\{1-\cos(s\,\Delta t/\tau_*)\}}{s^2\left\{1+(\ell_\circ/(U\tau_*))^2\right\}}\,\mathrm{d}s \\
&= \frac{\Delta^2}{12}\,\mathcal{F}\!\left(\frac{\Delta t}{\tau_*},\frac{U\,\Delta t}{\ell_\circ}\right),
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
\mathcal{F}(p,q) &= \frac{1}{2}\{(1+p)-|1-p|\} \\
&\quad -\frac{p}{q}\left\{1-\mathrm{e}^{-q/p}-\mathrm{e}^{-q}+\frac{1}{2}\mathrm{e}^{-q(1+p)/p}+\frac{1}{2}\mathrm{e}^{-q|1-p|/p}\right\}.
\end{aligned} \tag{31}$$

It is possible to express $p=\Delta t/\tau_*$ in terms of ordinary observable parameters. First we find an expression for τ_* by means of (27), (28), (4), and (22):

$$\tau_*^2 = \frac{2\Delta^2}{\langle\dot{u}^2\rangle} = B^2\left\{\langle\dot{u}^2\rangle\Delta t^2/2\right\}^{-1} = B^2\left\{\frac{\alpha_1}{\kappa^{2/3}}u_*^2\,(\ell_\circ/z)^{2/3}\,\mathcal{I}(U\,\Delta t/\ell_\circ)\right\}^{-1}. \tag{32}$$

Then, by means of (20), we obtain

$$p = \underbrace{\sqrt{\alpha_1}\kappa^{2/3}}_{\simeq 0.22}\,\frac{\ell_\circ}{B}\,\frac{(\ell_\circ/z)^{1/3}}{\ln(z/z_\circ)}\,\frac{U\,\Delta t}{\ell_\circ}\,\mathcal{I}^{1/2}\!\left(\frac{U\,\Delta t}{\ell_\circ}\right). \tag{33}$$

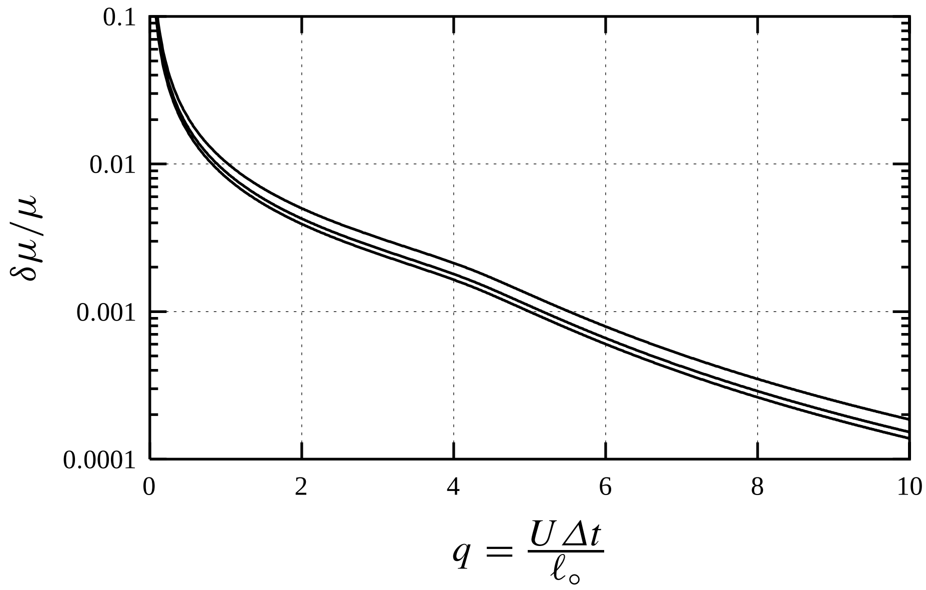

Figure 2: The bias of the dimensionless gust due to signal quantization for the Risø model P2546 with the distance constant $\ell_\circ = 1.8$ m and the calibration constant $B = 0.62$ m. The height is $z = 10$ m over a homogeneous terrain with the roughness length $z_\circ = 0.05$ m. Neutral stratification is assumed. The averaging time T is 10 minutes and the three curves represent $U =$ 5, 10, and 15 m s^{-1}, where the top curve corresponds to the lowest mean-wind speed.

Going back to (14), we see that the corresponding relative change in the gust factor becomes

$$\frac{\delta\mu}{\mu} = \frac{1}{2\mu^2}\,\frac{\delta\langle\dot{u}^2\rangle}{\langle\dot{u}^2\rangle}, \tag{34}$$

displayed in Fig. 2 for the Risø model P2546 with the distance constant $\ell_\circ = 1.8$ m and the calibration constant $B = 0.62$ m, in a neutrally stratified atmospheric surface layer, at the height $z = 10$ m in a homogeneous terrain with the roughness length $z_\circ = 0.05$ m.

Disjunct-Sampling Bias

In this section we neglect the quantization and assume that the wind speed is continuous. The classical Rice approach already discussed in a preceding section must be generalized to signal sampling with a finite time interval Δt between observations.

Let $u_0 = u(t)$ and $u_1 = u(t + \Delta t)$, and let $\mathcal{P}(u_0 < \mathcal{U}, u_1 > \mathcal{U})$ be the joint probability that $u_0 < \mathcal{U}$ and $u_1 > \mathcal{U}$. Then the lower limit for the average number that $\mathcal{U}$ has been exceeded in the period $T = N\Delta t$, where N is the number of observations in the period T, is $N \times \mathcal{P}(u_0 < \mathcal{U}, u_1 > \mathcal{U})$. The corresponding average rate of up-crossings can thus be written

$$\eta'(\mathcal{U}) = \frac{N}{T}\mathcal{P}(u_0 < \mathcal{U}, u_1 > \mathcal{U}) = \frac{1}{\Delta t}\mathcal{P}(u_0 < \mathcal{U}, u_1 > \mathcal{U}). \tag{35}$$

This equation can be reformulated in terms of the joint probability $\widetilde{p}(u_0, u_1)$ of u_0 and u_1:

$$\eta'(\mathcal{U}) = \frac{1}{\Delta t}\int_{-\infty}^{\mathcal{U}} \mathrm{d}u_0 \int_{\mathcal{U}}^{\infty} \mathrm{d}u_1\ \widetilde{p}(u_0, u_1). \tag{36}$$

Introducing new integration variables by

$$\left.\begin{aligned} u_c &= \frac{u_0+u_1}{2} \\ \Delta u &= u_1 - u_0 \end{aligned}\right\} \Longleftrightarrow \left\{\begin{aligned} u_0 &= u_c - \Delta u/2 \\ u_1 &= u_c + \Delta u/2 \end{aligned}\right. \tag{37}$$

we reformulate (36) to become

$$\eta'(\mathcal{U}) = \frac{1}{\Delta t}\int_0^{\infty} \mathrm{d}\Delta u \int_{\mathcal{U}-\Delta u/2}^{\mathcal{U}-\Delta u/2} \mathrm{d}u_c\, p_D(u_c, \Delta u) \simeq \frac{1}{\Delta t}\int_0^{\infty} \Delta u\ p_D(\mathcal{U}, \Delta u)\mathrm{d}\Delta u, \tag{38}$$

where

$$p_D(u_c, \Delta u) \equiv \widetilde{p}(u_c - \Delta u/2, u_c - \Delta u/2). \tag{39}$$

The equation (38) is analogous to (7). The variable transformation shows that u_c with the mean value U and Δu are uncorrelated and the assumption that u is Gaussian implies that

$$p_D(u, \Delta u) = \frac{1}{2\pi}\frac{1}{\sigma_u\,\langle \Delta u^2\rangle^{1/2}}\exp\left(-\frac{(u-U)^2}{2\sigma_u^2} - \frac{\Delta u^2}{2\langle \Delta u^2\rangle}\right). \tag{40}$$

Inserting in (38) we get

$$\mu'^2 = 2\ln\left(\frac{1}{2\pi}\frac{\langle \Delta u^2\rangle^{1/2}}{\sigma_u}\frac{T}{\Delta t}\right). \tag{41}$$

With (15) we get the equation analogous to (16):

$$\begin{aligned}
\langle \Delta u^2 \rangle &= \langle (u(t+\Delta t) - u(t))^2 \rangle \\
&= 2 \int_0^\infty [1 - \cos(\omega \Delta t)]\, S(\omega)\, \mathrm{d}\omega \\
&= 2 \int_0^\infty [1 - \cos(\omega \Delta t)]\, \frac{S_\circ(\omega)}{1 + (\ell_\circ\, \omega / U)^2}\, \frac{\sin^2(\omega \Delta t/2)}{(\omega \Delta t/2)^2}\, \mathrm{d}\omega \\
&= \frac{4}{\Delta t^2} \int_0^\infty \frac{S_\circ(\omega)}{1 + (\ell_\circ\, \omega / U)^2}\, [1 - \cos(\omega \Delta t)]^2\, \frac{\mathrm{d}\omega}{\omega^2}.
\end{aligned} \tag{42}$$

Using again (17), (42) can be written

$$\langle \Delta u^2 \rangle = 2\alpha_1\, (\varepsilon \ell_\circ)^{2/3} \mathcal{J}\left(\frac{U\, \Delta t}{\ell_\circ} \right), \tag{43}$$

where

$$\begin{aligned}
\mathcal{J}(q) &= \frac{2}{q^2} \int_0^\infty \frac{[1 - \cos(qs)]^2}{1 + s^2}\, s^{-11/3}\, \mathrm{d}s \\
&= \frac{8\pi \sinh^4(q/2)}{\sqrt{3} q^2} + \frac{243}{6160}\, \Gamma\left(\frac{1}{3}\right) \left\{ q^{8/3}\, {}_1F_2\left(1; \frac{17}{6}, \frac{10}{3}, \frac{q^2}{4}\right) \right. \\
&\quad \left. - (2q)^{8/3}\, {}_1F_2\left(1; \frac{17}{6}, \frac{10}{3}, q^2\right) \right\}.
\end{aligned} \tag{44}$$

With the last two equations and (18) and (19) we obtain the following relation between μ'^2 and μ^2, given by (41) and (14), respectively

$$\mu'^2 - \mu^2 = \ln\left(\frac{\langle \Delta u^2 \rangle}{\langle \dot{u}^2 \Delta t^2 \rangle} \right) = \ln\left(\frac{\mathcal{J}(U\, \Delta t/\ell_\circ)}{\mathcal{I}(U\, \Delta t/\ell_\circ)} \right). \tag{45}$$

The difference (45) is a function of the single parameter $q = U\, \Delta t/\ell_\circ$ and is shown in Fig. 3.

Inserting (24) into (45) we obtain

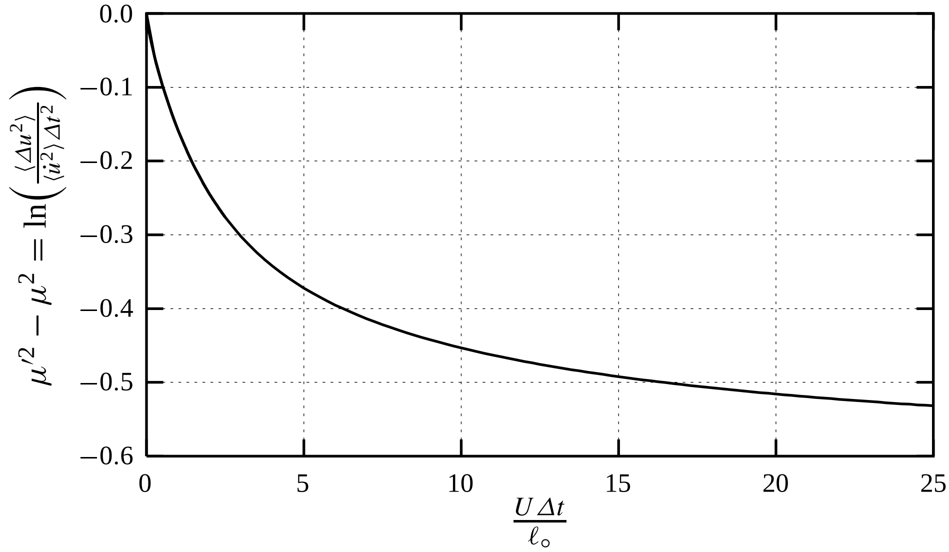

Figure 3: The difference $\mu'^2 - \mu^2$ as a function of $q = U\Delta t/\ell_\circ$, where U is the mean-wind speed, $\Delta(t)$ the time between recordings, and $\ell_\circ$ the cup-anemometer distance constant.

$$\mu'^2 = C + \frac{2}{3}\ln\left(\frac{\ell_\circ}{z}\right) + 2\ln\left(\frac{UT}{\ell_\circ}\right) + \ln\left(\frac{\mathcal{J}\left(\frac{U\Delta t}{\ell_\circ}\right)}{\left(\frac{U\Delta t}{\ell_\circ}\right)^2}\right). \tag{46}$$

By means of the last equation and (24) it is possible to estimate the negative, relative bias on the gust factor $(\mu' - \mu)/\mu$ in terms of the two parameters $UT/\ell_\circ$ and $U\Delta t/\ell_\circ$. This is illustrated in Fig. 4, where the relative bias loss is shown as a function of $U\Delta t/\ell_\circ$ for three values of the mean-wind speed U, and where T =10 min=600 s and $\ell_\circ$ =1,8 m (Risø model P2546).

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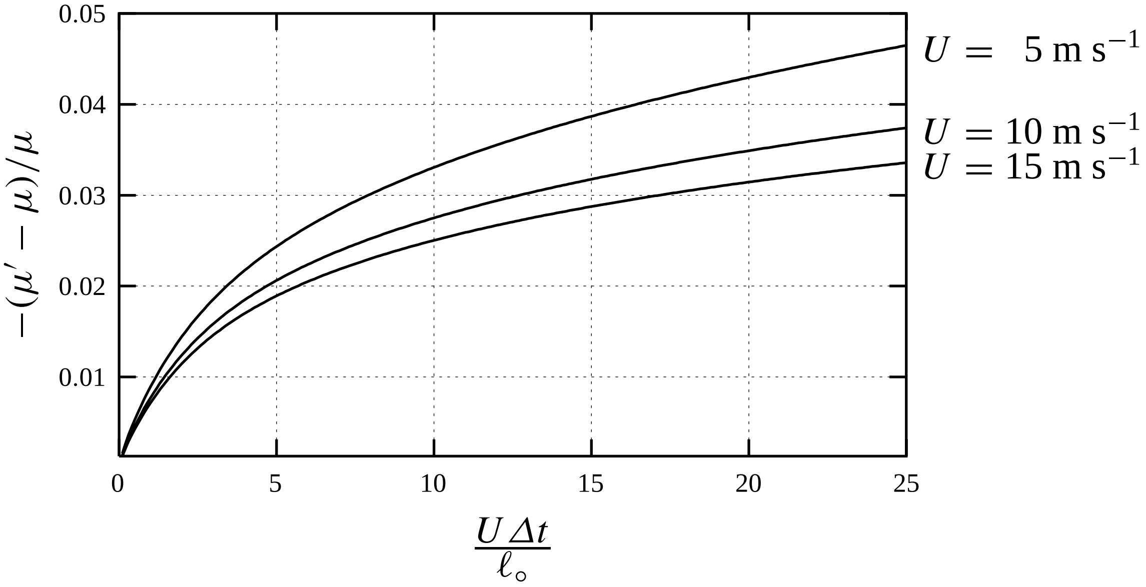

Figure 4: The relative bias loss as a function of $U\,\Delta t/\ell_\circ$ for U =5, 10, and 15 m s^{-1} for the Risø model P2546 with the distance constant $\ell_\circ$ =1.8 m.

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