Deficit of Wind-Speed Variance and Standard Deviation as measured by a Cup Anemometer

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The cup anemometer is a first-order line filter in the direction of the mean wind speed U with the distance constant $\ell_\circ$. This means that there is as reduction of the measured variance due to this filtering. In the limit $\ell_\circ \to 0$ this reduction goes to zero and it increases with $\ell_\circ$. We want to determine the loss of variance and standard deviation as functions of $\ell_\circ$.

The true variance in the flow direction $\sigma_\circ^2$ is equal to the area under the spectrum $F(k)$, where k is the wave number. We have

$$\sigma_\circ^2 = \int_{-\infty}^{\infty} F(k)\,\mathrm{d}k. \tag{1}$$

As discussed by Kristensen (1998), the cup anemometer can be considered a spatial, one-dimensional low-pass filter with the transfer function

$$H_{\mathrm{lp}}(\ell_\circ, k) = \frac{1}{1 + k^2\ell_\circ^2}. \tag{2}$$

The measured variance is then

$$\sigma^2 = \int_{-\infty}^{\infty} H_{\mathrm{lp}}(\ell_\circ, k) F(k)\,\mathrm{d}k = \int_{-\infty}^{\infty} \frac{F(k)\,\mathrm{d}k}{1 + \ell_\circ^2 k^2}. \tag{3}$$

We see that the reduction is most pronounced at large wave numbers and this suggests that we reformulate (3) as follows

$$\sigma^2 = \sigma_\circ^2 - \int_{-\infty}^{\infty} H_{\mathrm{hp}} F(k)\,\mathrm{d}k, \tag{4}$$

where

$$H_{\mathrm{hp}}(\ell_\circ, k) = 1 - H_{\mathrm{lp}}(\ell_\circ, k) = \frac{\ell_\circ^2 k^2}{1 + \ell_\circ^2 k^2} \tag{5}$$

is the corresponding high-pass filter which accounts for the "loss" of variance from the small eddies. It is this lack of resolution by the anemometer which causes the deficit of the measured variance.

The detailed behavior of $F(k)$ at low wave numbers is unimportant and we need to specify it in the inertial subrange where there is local isotropy. In this domain we use the standard formulation

$$F(k) = \frac{\alpha_1}{2}\varepsilon^{2/3}k^{-5/3}, \tag{6}$$

where ε is the rate of dissipation of specific kinetic energy (or just the dissipation) and $\alpha_1 \simeq 0.56$ the Kolmogorov constant for the streamwise velocity fluctuations (Kristensen et al. 1989). Inserting (6) in (4) we get

$$\sigma^2 = \sigma_\circ^2 - \frac{\pi\alpha_1}{\sqrt{3}}(\varepsilon\ell_\circ)^{2/3}. \tag{7}$$

This means that the loss of variance can be estimated by

$$\Delta[\sigma^2] \equiv \sigma_\circ^2 - \sigma^2 = \frac{\pi\alpha_1}{\sqrt{3}}(\varepsilon\ell_\circ)^{2/3}. \tag{8}$$

If we were interested in the loss of standard deviation we get the approximate expression from (7)

$$\sigma = \left[\sigma_\circ^2 - \frac{\pi\alpha_1}{\sqrt{3}}(\varepsilon\ell_\circ)^{2/3}\right]^{1/2} \simeq \sigma_\circ - \frac{\pi\alpha_1}{2\sqrt{3}}\frac{(\varepsilon\ell_\circ)^{2/3}}{\sigma_\circ} \tag{9}$$

so that the loss becomes

$$\Delta[\sigma] \equiv \sigma_\circ - \sigma = \frac{\pi\alpha_1}{2\sqrt{3}}\frac{(\varepsilon\ell_\circ)^{2/3}}{\sigma_\circ}. \tag{10}$$

The dissipation can be estimated quite accurately in the neutral atmospheric surface layer (strong winds). We have

$$\varepsilon = \frac{u_*^3}{\kappa z}, \tag{11}$$

where $\kappa \simeq 0.4$ is the von Kármán constant, z the measuring height, and u_* the friction velocity. We see that we must specify the friction velocity u_* and also $\sigma_\circ$, in the case of loss of standard deviation. There is an empirical relation between the standard deviation $\sigma_\circ$ and the friction velocity. According to Panofsky & Dutton (1984, p. 160)

Table 1: Loss of variances and of standard deviations by cup anemometers and a sonic at z = 10 m.

	$\ell_\circ$ (m)	$\Delta[\sigma^2]/\sigma_\circ^2$	$\Delta[\sigma]/\sigma_\circ$
Typical cup anemometer	3.9	17%	9%
WindsSensor P2546	1.8	10%	5%
Solent sonic	0.05	4%	2%

$$\frac{\sigma_\circ}{u_*} = 2.39 \pm 0.03 \tag{12}$$

We can now quantify the relative loss of variance and of standard deviation by

$$\frac{\Delta[\sigma^2]}{\sigma_\circ^2} = \frac{\pi\alpha_1}{\sqrt{3}\,\kappa^{2/3}}\left(\frac{u_*}{\sigma_\circ}\right)^2\left(\frac{\ell_\circ}{z}\right)^{2/3} \simeq 0.327\left(\frac{\ell_\circ}{z}\right)^{2/3} \tag{13}$$

and

$$\frac{\Delta[\sigma]}{\sigma_\circ} = \frac{\pi\alpha_1}{2\sqrt{3}\,\kappa^{2/3}}\left(\frac{u_*}{\sigma_\circ}\right)^2\left(\frac{\ell_\circ}{z}\right)^{2/3} \simeq 0.164\left(\frac{\ell_\circ}{z}\right)^{2/3}. \tag{14}$$

Let us, as an example, consider a cup anemometer (WindSensor, P2546) with $\ell_\circ = 1.8$ m. We get $\Delta[\sigma^2]/\sigma_\circ^2 \approx 10\%$ and $\Delta[\sigma]/\sigma_\circ \approx 5\%$.

The theory outlined here is based on the fortunate fact that we need to consider only the short-waved part of the turbulence. The long-waved/low wave-number part of the spectrum $F(k)$ is not well known. But all cup anemometers that work at all will, irrespective of their distance constants, measure the same velocities at long wavelength. Therefore the loss of variance for different anemometers can be easily related to their distance constants and the measuring height z. An anemometer with a high spatial resolution is the sonic anemometer, which measures the velocity along three acoustic 15 cm paths to obtain the wind velocity in all three dimensions. We have estimated that the transfer function for this sonic corresponds to a cup anemometer with a distance constant of 0.05 m.

The losses of variance and of standard deviation at z = 10 m are given in Table 1.

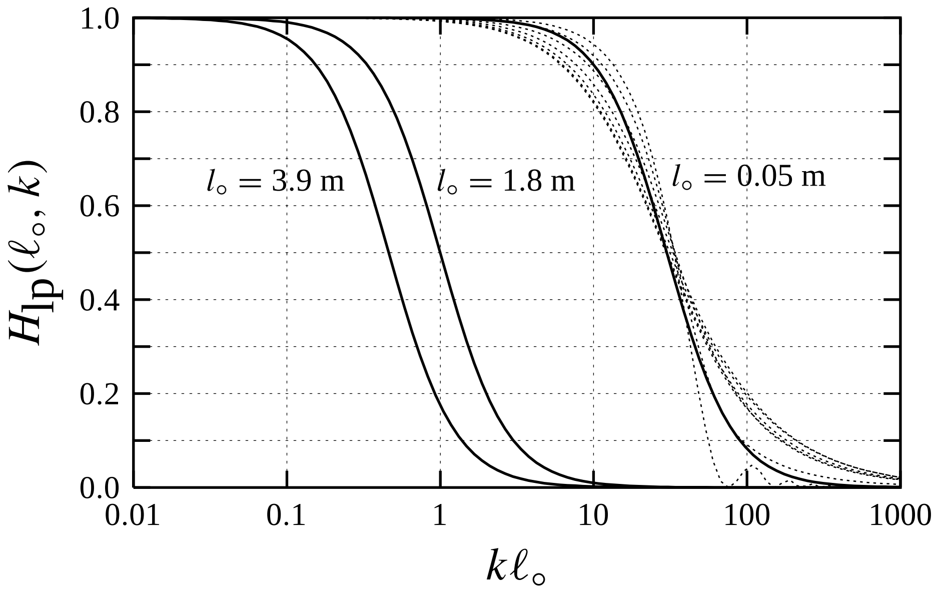

Figure 1: The transfer functions for the WindSensor P2546 cup anemometer with the distance constant $\ell_\circ$=1.8 m and a typical cup anemometer with $\ell_\circ = 3.9$ m. These are shown with solid lines together with an eye-ball fit to the many dotted-line sonic transfer functions, where each curve corresponds to the angle between the sonic path and the mean wind. An eye-ball fit to the mean suggests that $\ell_\circ = 0.05$ m. The abscissa is made dimensionless by means of the distance constant of the WindSensor P2546 cup anemometer.

References

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