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Research Article

The Perennial Cup Anemometer

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Key words: calibration; starting speed; calibration distance; angular response; distance constant; overspeeding

A short version of the history of the cup anemometer precedes a more technical discussion of the special features of this instrument. These include its extremely linear calibration and the non-linearity of its response to wind speed changes. A simple conceptual model by Schrenk is used to demonstrate this and to explain why the cup anemometer is able to start from a zero rotation rate at zero wind to one corresponding to a sudden change in the ambient wind speed to a finite value. The same model is used to show that the cup anemometer should be characterized by a distance constant rather than by a time constant. The bias in the measured mean wind speed due to the random variations in the three velocity components is discussed in terms of standard, semi-quantitative turbulence models, and the main thesis is that this bias is overwhelmingly dominated by the fluctuations of the lateral wind velocity component, i.e. the wind component perpendicular to the mean wind direction, and not, as is often assumed, by the longitudinal wind velocity component. It is shown theoretically and tested experimentally that the bias due to lateral wind velocity fluctuations can be significantly reduced by means of a special data processing of the simultaneous signals from a cup anemometer and a wind vane. This means that, with care, the overall overspeeding *can be reduced to less than 1%. Copyright ©1999 John Wiley & Sons, Ltd.*

Introduction

The cup anemometer is probably the most common instrument for measuring the wind speed at places where weather observations are routinely carried out. We see them in airports, at wind farms and at construction sites—and for good reasons. Experience has shown that the cup anemometer is a robust and reliable instrument which can operate unattended for years. It is easy to install because it is omnidirectional, as Figure 1 shows. Finally, the rotor rotation rate S is very close to being proportional to the wind speed U, i.e. the calibration is almost linear and easy to determine with a high degree of accuracy.

In contrast, the Pitot tube which is now mostly used on aircraft has a very non-linear calibration as its output signal, a pressure difference, is proportional to the square of U. This instrument is mostly used on aircraft where the airspeed is so large that signal linearization provides sufficiently precise measurements, or in wind tunnels where the calm laboratory environments make it possible to measure small pressure differences very accurately.

The cup anemometer was invented by the Irish astronomer T. R. Robinson in 1846[1,2] and is still, as more or less the same construction, in use today. The literature shows that until the end of the 1920s the users were mostly interested in the linearity of the calibration. Brazier[3] showed that the shorter the rotor radius, i.e. the distance r from the axis of rotation to the centre of a cup, the better is the linearity. This observation was confirmed by Patterson[4] who, for a number of cup anemometers, determined the so-called *anemometer factor*

$$f = \frac{U}{rS} \tag{1}$$

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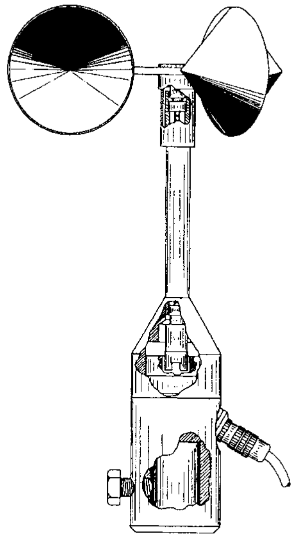

Figure 1. Cup anemometer model Risø-70. The height is 0·27 m, the rotor radius r *0·058 m and the diameter of the conical cup 0·07 m. The rotor, made of carbon-reinforced plastic, weighs 0·04 kg*

His investigations showed that the 'constant' f varied between 2·5 and 3·5 from instrument to instrument. This was not a big surprise at that time. In fact, the inventor Robinson believed that for an ideal anemometer without friction in the bearings the factor would be exactly 3 and that he had discovered a law of nature. He had tried to calibrate his instrument by mounting it on a wagon and then, in calm weather, driving this wagon with a constant, well-known speed. Counting the number of rotor revolutions over a certain time, he could determine S and, since U was given by the speed of the wagon, he could calculate f. Patterson used a wind tunnel and obtained much more accurate results.

In the 1920s the interest in the function of the cup anemometer turned from the linearity of the calibration towards its *overspeeding*. This phenomenon, which has its root in the asymmetric response to changes in wind speed, is quantitatively the positive bias in the measurement of the mean wind speed in turbulent environments, where the rotor will respond more readily to an increase than to a decrease in the wind speed. Kaganov and Yaglom[5] and Wyngaard[2] have given interesting accounts of the many attempts to analyse overspeeding. According to Kaganov and Yaglom,[5] Sabinin was the first to analyse the problem, by calculating the cup anemometer response to synthetic turbulence consisting of a periodically varying wind speed. Wyngaard,[2] however, believes that the first serious attempt to understand overspeeding is due to Schrenk[6] who derived a model dynamic equation for the cup anemometer motion. The struggle to obtain a formula for the response to real turbulence continued unsettled until Kaganov and Yaglom[5] and, simultaneously, Busch and Kristensen[7] solved the problem by means of a second-order perturbation calculation.*

After the mid-1970s the discussion about overspeeding lost its momentum. This moratorium lasted until the wind turbine industry by the end of the 1980s started requiring extreme accuracy in the determination of the mean wind speed. The reason was that the horizontal flux of wind energy is approximately proportional to the mean wind speed to the third power, so that an error in the measured mean wind speed of about 3% would result in an error of about 10% in the estimated wind power. Since wind turbines are often sold with a certain guaranteed annual production, we are now in an area where money is doing the talking—amongst manufacturers, customers and roaming lawyers. This has given a new opportunity for making efforts to obtain a better understanding of cup anemometer dynamics.

*It should be added that A. M. Yaglom, according to Kaganov and Yaglom,[5] solved the problem already in 1954.[8] However, his revelation was communicated in Russian and, consequently, probably unknown in the western world.

In the following we shall reiterate aspects of the dynamics of the cup anemometer and answer the question about how precisely it is possible to use it for the determination of the mean wind speed in turbulent environments. We show experimental data to support these answers.

Cup Anemometer Dynamics

Let us first consider the calibration. The standard procedure is to use a wind tunnel with a flat wind profile and operate it at a number of wind speeds U. We want to determine the relation between the angular velocity S of the rotor and U. However, since we suspect that turbulence has an influence, we must specify the turbulence characteristics. In general, such general specifications are rather complicated and it would also be difficult to obtain general agreement on the appropriate scale and intensity of the turbulence for calibration purposes. Consequently, the ideal wind tunnel would have a laminar flow. This means that we will choose low-turbulence wind tunnels with very thin boundary layers for calibration purposes.

Now there are only the two quantities U and S to consider. The first, the wind speed in the wind tunnel, is measured by means of a precision Pitot tube, while S is determined by counting the number of rotations over a certain time. Letting U vary from $0\,\mathrm{m\,s^{-1}}$ to perhaps as large a speed as $80\,\mathrm{m\,s^{-1}}$, we can relate S to U. A plot of S versus U might, somewhat ideally, look like the sketch in Figure 2.

Owing to the friction in the bearings, a certain wind speed is necessary to start the cup anemometer. In this wind speed domain the calibration is quite complicated. In fact, this 'starting' speed is not in general equal to the 'stopping' speed. This means that when the wind speed is low, i.e. below about $1\,\mathrm{m\,s^{-1}}$, the calibration become unreliable. On the other hand, when U is about $2\,\mathrm{m\,s^{-1}}$ or more, the calibration for a reasonably good cup can be described by just two parameters, a fictitious *starting speed* U_0 and the *calibration distance* l. One will find that in this domain the linear relation

$$S = \frac{U - U_0}{l} \tag{2}$$

will be accurate within $0{\cdot}1\,\mathrm{m\,s^{-1}}$ for several years in active duty.

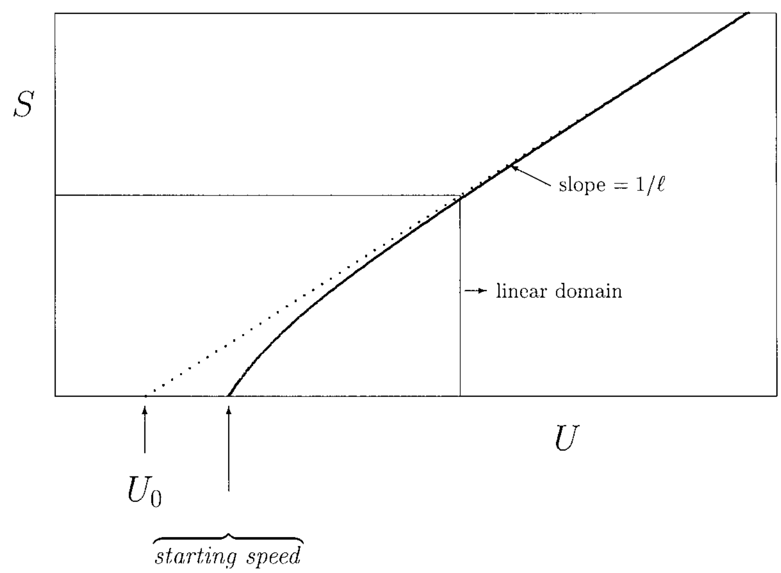

Figure 2. Calibration curve

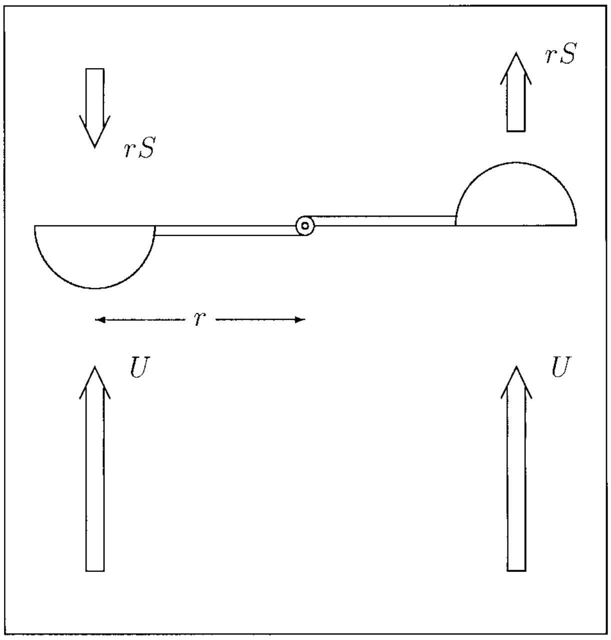

Figure 3. Schematic diagram of wind torque on cup–anemometer rotor

Often the parameter U_0 is so small compared with the actual wind speed that it may be ignored in expression (2). This means that l can be interpreted as the length of the column of air that has to go through the anemometer to make it turn 1 rad. A comparison between (1) and (2) shows that in this case l is about $f \times r \approx 3 \times r$. This is pure kinematics: when the friction in the bearings can be neglected, the calibration does not depend on the mass of the rotor, only on the rotor geometry.

To better understand what is actually happening, we now apply the simple model used by Schrenk.[6] This model is based on the simple fact that the drag on a cup is larger when the wind blows into the cup than when it blows on its backside. Figure 3 shows a top view of a simple two-cup anemometer. It turns clockwise such that the right-hand cup, moving with the wind, 'feels' a wind speed equal to $U - rS$, while the left-hand cup, moving against the wind, 'experiences' the wind speed $U + rS$. The drag on a cup is proportional to the square of the relative wind speed, to the air density ρ_a and to the cup area. The drag coefficient will consequently depend on whether the wind blows into the cup or on the backside. The former, C_+, will accordingly be larger than the latter, C_-.

Each cup provides a torque equal to the drag force multiplied by the arm r, which is the same for both cups. We may therefore write the equation of motion as

$$I\frac{\mathrm{d}S}{\mathrm{d}t} = \mathcal{M} = r[K_+(U - rS)^2 - K_-(U + rS)^2] \tag{3}$$

where the dependence on the density of the air, the cup area and the drag coefficients are combined in the coefficients K_+ and K_-, and I is the rotor moment of inertia. The equation states Newton's equation in the form where angular moment, equal to the moment of inertia times the angular acceleration, is proportional to torque $\mathcal{M}$.

In a wind tunnel where U is constant, the anemometer has a constant angular velocity S, so in this situation the left-hand side of (3) is zero. This is possible because

$$K_+ > K_- \tag{4}$$

Solving for S, we see that it is proportional to U and that the 'historical' anemometer factor* becomes

$$f = \frac{U}{rS} = \frac{\sqrt{K_+} + \sqrt{K_-}}{\sqrt{K_+} - \sqrt{K_-}} \tag{5}$$

As K_+ and K_- are both proportional to the density of the air, this model implies that for an anemometer with frictionless bearings the calibration does not depend on the air temperature or pressure. This is consistent with the observation that the calibration for all practical purposes is dependent only on the cup geometry.

We will now analyse what happens if the wind speed is suddenly changed from U to $U + u$. Initially S is unchanged, but its time derivative is no longer zero. We have

$$\begin{aligned} I\frac{\mathrm{d}S}{\mathrm{d}t} &= r[K_+(U + u - rS)^2 - K_-(U + u + rS)^2] \\ &= \underbrace{r[K_+(U - rS)^2 - K_-(U + rS)^2]}_{=0} \\ &\quad + 2r[K_+(U - rS) - K_-(U + rS)]u + r(K_+ - K_-)u^2 \end{aligned} \tag{6}$$

The first term on the right-hand side is obviously zero, so that

$$\sqrt{K_+}(U - rS) = \sqrt{K_-}(U + rS) \tag{7}$$

Together with inequality (4), this equation implies

$$K_+(U - rS) > K_-(U + rS) \tag{8}$$

In other words, the coefficients of u and u^2 are both positive. This means that if u is positive, then $\mathcal{M}$ and the derivative of S are positive. If u is negative, $\mathcal{M}$ will be negative too,† so that the derivative of S becomes negative. However, owing to the quadratic term, this derivative will be numerically smaller than in the case where u is positive and of the same magnitude. This simple model can, in other words, provide a kind of quantitative explanation of why a cup anemometer accelerates more readily than it brakes and thus spends more time on the high side than on the low side of the mean in a turbulent wind. This response asymmetry, which might be perceived as a less desirable property, is, on the other hand, a necessary condition for the cup anemometer to start rotating when there is a wind. This can easily be seen by inspecting (6) in a situation where both U and S are zero, i.e. the rotor is not moving, and then, all of a sudden, there is a breeze of magnitude u. The only term on the right-hand side of (6) is the quadratic term, which will deliver a positive torque and start the rotor.

Let us now take a step further and let time pass and include the response s to the change u. Both these quantities are small, so we neglect second-order terms. Now the equation of motion becomes

$$\begin{aligned} I\frac{\mathrm{d}S}{\mathrm{d}t} &= r[K_+(U + u - rS - rs)^2 - K_-(U + u + rS + rs)^2] \\ &\approx 2r[K_+(U - rS) - K_-(U + rS)]u \\ &\quad - 2r^2[K_+(U - rS) + K_-(U + rS)]s \end{aligned} \tag{9}$$

We observe that the coefficient of s is negative, so that if u is positive, causing S to start to grow, then this growth by itself will lead to braking. This prevents an exponential growth of the rotation rate. An

*The other solution is not realistic; it states that the cups of the rotor move faster than the wind.

†It seems conceivable that u could be numerically so large that the quadratic term would be dominating in (6). A closer look will show, however, that in this case $|u| > 4U\sqrt{K_+K_-}/(\sqrt{K_+} + \sqrt{K_-})^2$. An anemometer factor of about $f = U/(rS) = 3$ then implies that $|u|$ must be larger than $32U/9 \gtrsim 3U$, which is unrealistic.

equilibrium will be attained corresponding to the new constant wind speed $U + u$. The larger the coefficient of s, the faster this equilibrium is established. In fact, this coefficient of s, divided by the moment of inertia I, has the dimension of a reciprocal time—at time τ_0, which could be termed a *time constant*. Mathematically, τ_0 is the time it takes to reach $1 - e^{-1} \approx 63\%$ of the final rotation rate corresponding to the wind speed $U + u$. We note that τ_0 is inversely proportional to $U \propto S$, so that the length scale

$$\begin{aligned} l_0 = U\tau_0 &= \frac{IU}{2r^2[K_+(U - rS) + K_-(u + rS)]} \\ &= \frac{If}{2r^2[K_+(f-1) + K_-(f+1)]} \end{aligned} \tag{10}$$

is independent of the wind speed. This length scale is—just like the calibration distance l—an instrument constant. It is called the *distance constant* l_0. It is a true instrument constant and a better alternative to τ_0 for describing how fast the instrument reacts to changes. l_0 is the length of the column of air that has to blow through the anemometer for this to reach 63% of its final reaction to a change.

We see that l_0 is proportional to the moment of inertia I, i.e. to the density of the rotor material, and to the arm r to the fifth power. K_+ and K_- are proportional to the area of a cup and to the air density ρ_a. Considering only geometrically similar cup anemometers, the cup area is proportional to r^2. This implies that l_0, just like l, is proportional to r. In contrast with l, however, l_0 is also proportional to the density ratio ρ/ρ_a. This means that rotors made of heavy material react more sluggishly than light-material rotors. It is also worth noticing that l_0 is inversely proportional to the air density ρ_a, which means that at higher altitudes the instrument might react 10%–20% more slowly than at the sea surface. (The calibration, on the other hand, is independent of the altitude, except that the friction in the bearings may cause a slight increase in l.) Most modern cup anemometers have distance constants from 1 to 2 m.

The simple model (3) is unfortunately not sufficient for describing everything. The first thing we notice is that it does not account for friction in the bearings. This, however, is not of the greatest concern, because when U is larger than about $2\,\mathrm{m\,s^{-1}}$, the anemometer is operating in the linear domain (see Figure 2) and we can account for the calibration by means of just one extra parameter, namely U_0. When defining the calibration length l (and the anemometer factor f) and the distance constant l_0, we just have to replace U by $U - U_0$.

There is another, more serious, problem which has to do with the fact that it only describes the dynamics in one spatial dimension; it is not taken into consideration that the turbulent wind varies not only in magnitude but also in direction and inclination (the vertical angle of attack). The model can be used to calculate the overspeeding in the sense we have discussed so far, but the errors in the measured mean wind speed which occur because of horizontal and vertical variations in the wind direction must also be determined, and here the model is of little use.

Let us analyse the consequence of the fact that the turbulent wind has variations in all three spatial directions. First we need a suitable co-ordinate system for the description of the wind velocity vector $\mathcal{U}$. We use the conventional co-ordinate system where the x-axis is aligned with the mean wind $\langle\mathcal{U}\rangle$, which is usually a 10 min average. We denote the magnitude of this vector as U. The fluctuating components are u along the x-axis, v along the horizontal y-axis and w along the vertical z-axis. Thus in this co-ordinate system we have

$$\mathcal{U} = \begin{Bmatrix} U \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \tag{11}$$

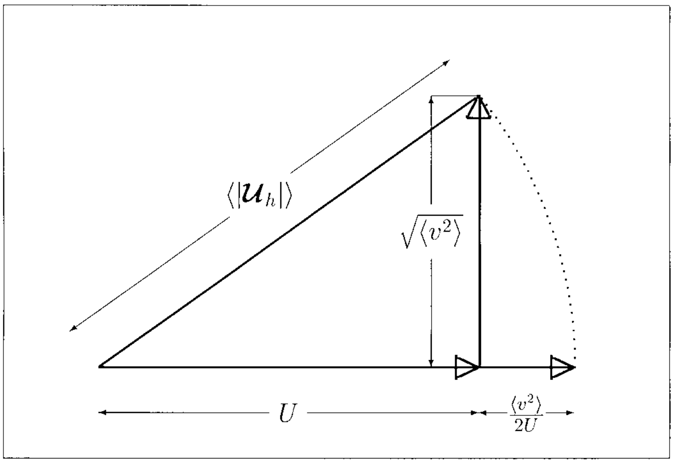

Figure 4. Graphical interpretation of DP error

An 'ideal' cup anemometer measures the horizontal projection of the total length of air which has passed through it during the averaging time. This means that the measured average from such an instrument becomes

$$\langle|\mathcal{U}_{\mathrm{h}}|\rangle = \left\langle\sqrt{(U+u)^2+v^2}\right\rangle \approx U + \frac{\langle v^2\rangle}{2U} \tag{12}$$

We observe that $\langle|\mathcal{U}_{\mathrm{h}}|\rangle$ is always larger than U. This is illustrated, somewhat exaggeratedly, in Figure 4. We measure $\langle|\mathbf{U}_{\mathrm{h}}|\rangle$ and not U, which is smaller by an amount equal to half the variance of the lateral velocity component divided by U. This 'error' is what MacCready[9] termed the *DP error*. It stands for data-processing error; in other words, an error in the signal processing. Perhaps it would be more to the point to call it a 'data-interpreting error'. The relative DP error, for which we will use the symbol δ_v, is obtained by dividing by U:

$$\delta_v = \frac{\langle v^2\rangle}{2U^2} \tag{13}$$

The ideal cup anemometer measures the mean of the horizontal speed, i.e. the mean of the magnitude of the horizontal projection of $\mathcal{U}$. Such an instrument has what we call a *cosine* response. In the real world, few cup anemometers are ideal, as illustrated in Figure 5. This means that there will usually be a systematic relative error δ_w on the determination of U. It is similar to the DP error, being proportional to the variance $\langle w^2\rangle$ of the vertical velocity component:

$$\delta_w = \mu_w \frac{\langle w^2\rangle}{2U^2} \tag{14}$$

Here the dimensionless instrument constant μ_w characterizes the deviation of the angular response $g(\vartheta)$ from $\cos(\vartheta)$. Detailed measurements carried out by Wyngaard *et al.*[10] and similar measurements on seven different cup anemometers by Coppin[11] show that μ_w can vary quite a lot from instrument to instrument, from $\mu_w = 0$ to 2.

Now we come to the error which is usually considered the overspeeding, namely the effect of the variation in u, the component of $\mathcal{U}$ along the mean wind direction. *A priori* we expect that the relative overspeeding δ_u, like δ_v and δ_w, is proportional to the variance $\langle u^2\rangle$. On the other hand, if we have a fast-responding cup anemometer with a small distance constant l_0, it will be able to react to changes in the

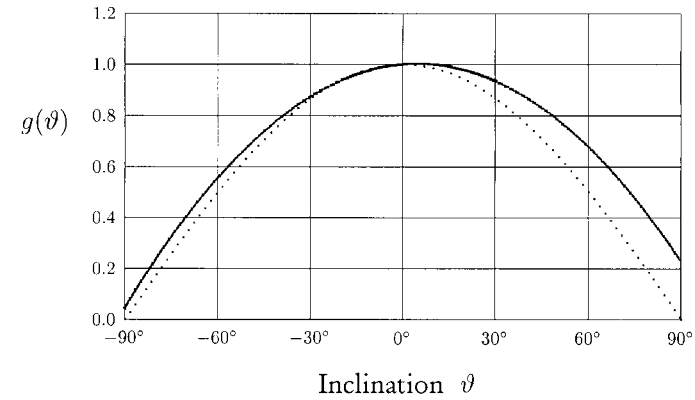

Figure 5. Typical cup anemometer response $g(\vartheta)$ *to inclination angle* ϑ. *The dotted line shows the ideal response, the cosine response*

wind speed if these changes are caused by eddies larger than l_0 which are carried to the anemometer by the mean wind. These large-scale changes will therefore be included correctly in the mean wind. The contribution to δ_u must come from eddies smaller than l_0. It will be natural to postulate that δ_u is proportional to some measure of the variation u over the distance l_0 in the mean wind direction. In other words,

$$\delta_u \propto \frac{\langle (u(x + l_0) - u(x))^2 \rangle}{U^2} \tag{15}$$

We see that the numerator in this expression is the so-called structure function and we know from Kolmogorov[12] that this function is proportional to $l_0^{2/3}$ as long as l_0 is small compared with the scale λ of the turbulence, i.e. with the magnitude of the largest eddies. Since the structure function is also proportional to $\langle u^2 \rangle$, we are led to the assumption that we may write the relative overspeeding in the form

$$\delta_u = \mu_u \frac{\langle u^2 \rangle}{2U^2} \left(\frac{l_0}{\lambda} \right)^{2/3} \tag{16}$$

The investigations by Wyngaard *et al.*[10] and Coppin[11] show that for all eight different cup anemometers the dimensionless constants μ_u have the same value, of the order unity, within 25%.*

We see that the turbulence intensities $\sqrt{\langle u^2 \rangle}/U$, $\sqrt{\langle v^2 \rangle}/U$ and $\sqrt{\langle w^2 \rangle}/U$ are key parameters when estimating the systematic errors on the measured mean wind speed. Usually $\langle u^2 \rangle$ and $\langle v^2 \rangle$ are about equal in magnitude and somewhat larger than $\langle w^2 \rangle$. Since l_0 is much smaller than λ in most situations, we conclude that δ_v must be the dominating error.

To be more specific, let us consider a cup anemometer at a height of 10 m over a plane grass surface. Here $\langle u^2 \rangle/(2U^2)$ and $\langle v^2 \rangle/(2U^2)$ will vary between 0·01 and 0·08 depending on the atmospheric conditions. At 40 m the corresponding numbers will be about half that. The length scale λ will typically range from 200 to 1000 m. Consequently, δ_v will be in the interval from 0·5% to 8%. δ_u, on the other hand, will hardly be larger than 1% under the same conditions. At heights of 10 and 40 m, $\langle w^2 \rangle/(2U^2)$ will be about 0·02 and 0·1 respectively. This means that δ_w will be between 1% and 2%.

Of course, one can be a little sceptical about the realism of the expressions (13), (14) and (16) for the relative systematic errors. It should be pointed out though that a much more rigorous analysis[13] leads to the same result.

We believe that we now understand how to quantify these errors with reasonable accuracy. However, in 1968 when the famous Kansas experiment was carried out, the situation was different. The results from

*The actual magnitude of μ_u depends on the precise definition of the turbulent length scale λ.

this experiment, which has been described so vividly by Kaimal and Wyngaard,[14] are now the basis for the wind code used by construction engineers. During the experiment it was found that unless all the cup anemometer measurements were reduced by 10%, it was impossible to make them consistent with the measurements from the sonic anemometers and the drag plate. At that time it was believed that the 10% bias was a δ_u error, but in the light of the present considerations it seems more likely that the bias was a δ_v or a DP error.

Cup Anemometer Design

The first cup anemometers had four cups. Robinson who invented the instrument in 1846 believed that this number of cups would be the minimum if the rotor were to turn evenly and thereby provide a linear calibration. Patterson[4] investigated this idea more closely and found that a two-cup rotor runs very unevenly, while a three-cup rotor in this respect is actually superior to a four-cup rotor and also has a more linear calibration. He found that three cups give a larger torque than four cups, probably because the cups in the latter case will be more in the wakes of the other cups. Today virtually all cup anemometers have three cups.

Patterson[4] also documented that a six-cup rotor in two layers, turned 60° with respect to each other, provides almost twice the torque of a three-cup rotor. It will reduce the starting speed U_0, because the friction in the bearings will become relatively less important. It is also possible to exploit the two-deck rotor design to obtain a better angular response by 'tuning' the vertical distance between the two decks. Thereby it is possible to reduce the δ_w error.

We saw that the distance constant l_0, which determines how fast the anemometer reacts to changes in the wind speed and therefore is also closely related to the 'classical' overspeeding δ_u, is proportional to the rotor radius r and the density of the rotor material. Consequently, we can reduce δ_u by making the anemometer small and light. Frenzen[15] constructed a two-deck, six-cup anemometer for field use with a distance constant as small as 0·25 m, and this means that such an anemometer, in regard to temporal resolution of a signal of the streamwise velocity component, can easily compete with a sonic anemometer.

It seems safe to postulate that it is possible to reduce δ_u and δ_w to insignificance by means of an appropriate design. The DP error which is due to wind direction fluctuations must be dealt with in a different way, as we shall see now.

Signal Analysis

It has already been pointed out that a column of air of length $2\pi l$ is required to blow through the rotor to make it turn one revolution. Counting the number N of revolutions over a certain time T, we get the mean wind speed by means of the equation

$$\langle|\mathcal{U}_\mathrm{h}|\rangle = \frac{N \times 2\pi l}{T} \tag{17}$$

However, as we saw, this leads to a systematic positive bias on the mean wind speed if this quantity is defined as the mean U of the horizontal component of the wind vector.

One possibility to reduce this DP error is to combine the signal from the anemometer with the signal from a wind vane measuring the wind direction. We simply use the anemometer signal for triggering a wind vane read-off: for every full rotor revolution we record the direction ϕ of the wind. If there are N revolutions in the period T and if the direction in a geographical co-ordinate system with the x-axis

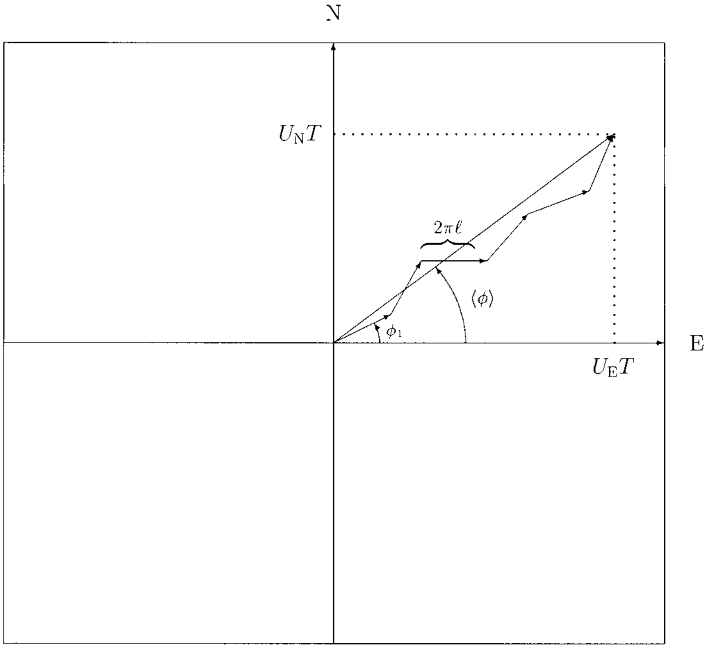

Figure 6. Schematic presentation of vector addition

pointing east and the y-axis pointing north is ϕ_i after period number i, then one can determine the horizontal mean wind vector by the equation

$$\begin{Bmatrix} U_{\mathrm{E}} \\ U_{\mathrm{N}} \end{Bmatrix} = \frac{2\pi l}{T} \begin{Bmatrix} \sum_{i=1}^{N} \cos(\phi_i) \\ \sum_{i=1}^{N} \sin(\phi_i) \end{Bmatrix} \tag{18}$$

We see immediately that (18) is a generalization of (17), since the two equations give the same result if the direction ϕ_i is constant. Figure 6 is a graphical representation of the procedure given by (18). We carry out a vector addition, which is easily done on-line with modern computer recording techniques.

We can now obtain a very precise determination of the magnitude U and the direction $\langle\phi\rangle$ of the horizontal component of the mean wind velocity vector from the equations

$$U = \sqrt{U_{\mathrm{E}}^2 + U_{\mathrm{N}}^2} \tag{19}$$

and

$$\langle\phi\rangle = \tan^{-1}\left(\frac{U_{\mathrm{N}}}{U_{\mathrm{E}}}\right) \tag{20}$$

Using this signal processing,* we have obtained a rather complete removal of the DP error,[13] but there are another two bonuses.

First, we have solved the standard problem in calculating the mean wind direction. This can be illustrated by the following example. Consider a situation where the direction oscillates around north,

*One must actually use the function $\tan^{-1}2(x, y)$ rather than $\tan^{-1}(y/x)$ in order to cover the entire compass without problems.

i.e. between a little less than 360° and a little more than 0°. Then a straightforward averaging of the direction may give any result, e.g. 180°, whereas (18) and (20) will give the correct mean wind direction.

The other advantage is that one can easily determine the wind direction variance, since

$$\langle v^2 \rangle = \langle |\mathcal{U}_{\mathrm{h}}| \rangle^2 - U^2 \tag{21}$$

which is easily seen by inspection of Figure 4.

Field Testing

Two types of experiments can be considered to test whether or not the DP error dominates the systematic mean wind error.

The first type is one in which we use only a cup anemometer and a wind vane to determine $\langle |\mathcal{U}_{\mathrm{h}}| \rangle$, U and $\langle v^2 \rangle$ by means of (17)–(20) and then use these quantities to see if (12) is fulfilled. This test is not very strong, since $\langle v^2 \rangle$ and $\langle |\mathcal{U}_{\mathrm{h}}| \rangle$ are not obtained by means of an independent anemometer. We can only use such measurements to test for inconsistencies and to illustrate how the scheme works.

In the other type of experiment, $\langle |\mathcal{U}_{\mathrm{h}}| \rangle$ is determined by means of another anemometer, e.g. a sonic.

As we shall see, the experiments do not contradict our thesis that in a turbulent atmosphere the DP error overwhelmingly dominates the positive bias. In fact, the scheme for calculating the DP error, developed in the previous section, seems to work well. It seems reasonable here to apply Ockham's razor[16] and say that we believe that the DP error is the *only* serious bias of the measured mean wind speed.

Cup and Vane

In November 1995 a small experiment was carried out at Risø National Laboratory. A combined cup–vane anemometer, shown in Figure 7, was mounted on a boom at a height of 11·2 m on the Risø tower. The instrumentation of the boom, which is pointing towards 225°, is shown in Figure 8.

The measurements were carried out from 00:05 on 15 November until 12:55 on 17 November. The 10 min averages were calculated according to (17)–(21) and the results are given in Figure 9.

Figure 9 shows almost an identity between $(\langle |\mathcal{U}_{\mathrm{h}}| \rangle - U)/U$ and $\langle v^2 \rangle/(2U^2)$. The exception is when the bias is large (30%–40%), as the bottom frame shows. This will usually correspond to rather low wind speeds where the RMS wind direction fluctuations are about 50° and where terms of higher order than $\langle v^2 \rangle$ start to play a role.

In Figure 10, $(\langle |\mathcal{U}_{\mathrm{h}}| \rangle - U)/U$ is plotted versus $\langle v^2 \rangle/(2U^2)$. A straight line is fitted to the data where the bias is less than 5%. We see that the least squares fit is almost perfect when the bias is less than about 10%. For higher values the second-order expansion (12) is apparently not accurate enough.

Cup, Vane and Sonic

For 4 days in July 1994 we measured the wind speed from two 6 m high masts inside an artillery shooting range at Borris in Jutland. Figure 11 shows the set-up.

The sampling rates were 10 Hz for the sonic and 5 Hz for the cup and the vane. The 10 min averages of the signals from the cup and the vane from four different days are shown in Figure 12.

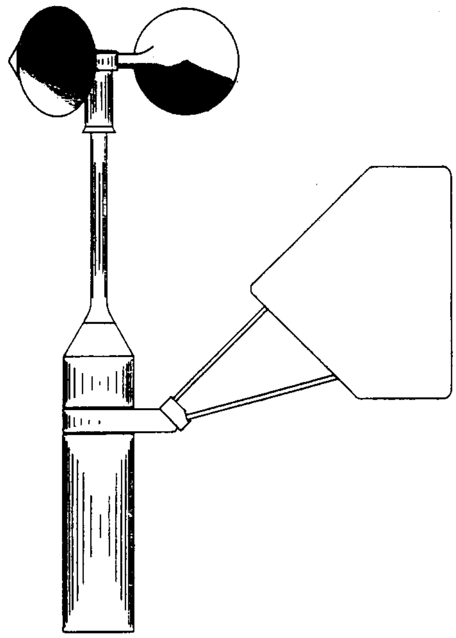

Figure 7. Risø combined cup–vane anemometer. The height of the instrument is 0·38 m from the base to the middle of the cup rotor. The rotor material is carbon-reinforced plastic. The instrument body is made of aluminium and the vane of hardened plastic-laminated styrofoam. The calibration distance l and the distance constant l_0 are about 0·2 m and 1·5 m respectively. The distance constant and the damping ratio for the vane are about 0·6 m and 0·4 respectively

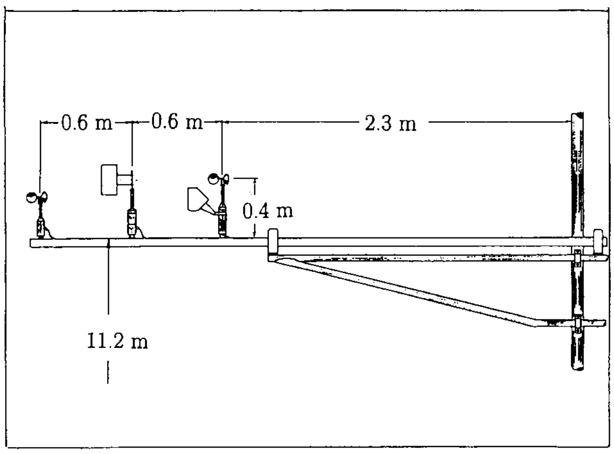

Figure 8. Experimental set-up. The cup–vane anemometer is mounted 2·3 m from the tower. The two other instruments further out on the boom, the vane and the cup anemometer, are not used in these measurements

Denoting the instantaneous horizontal components of the wind velocity measured by the sonic anemometer in the instrument's own Cartesian co-ordinate system as $\tilde{u}_l$ and $\tilde{v}_l$, we calculate the instantaneous speed

$$\tilde{s}_l = \sqrt{\tilde{u}_l^2 + \tilde{v}_l^2} \tag{22}$$

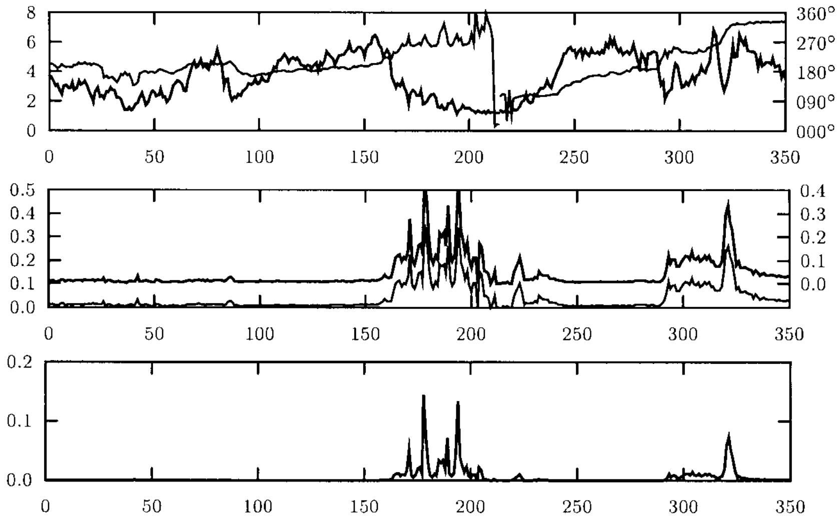

Figure 9. Time series of 10 min averages. The abscissae are the time in 10 min units. In the top frame the thick line is U *and the thin line the mean wind direction (270° − ⟨ϕ⟩). The wind blows through the tower when the direction is about 45°. This only occurs in a short period. The middle frame shows* $(\langle|\mathcal{U}_h|\rangle - \mathrm{U})/\mathrm{U}$ *(thick line) and* $\langle \mathrm{v}^2\rangle/(2\mathrm{U}^2)$ (thin line). In order to be able to discern these two curves, we have added 0·1 to the first. The bottom frame just shows the difference $(\langle|\mathcal{U}_h|\rangle - \mathrm{U})/\mathrm{U} - \langle \mathrm{v}^2\rangle/(2\mathrm{U}^2)$

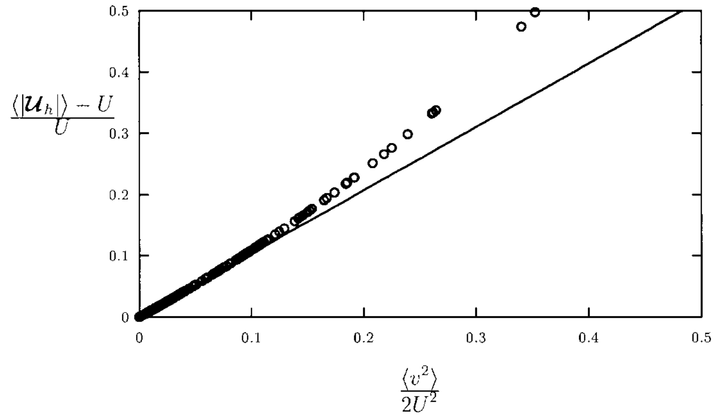

Figure 10. $(\langle|\mathcal{U}_h|\rangle - \mathrm{U})/\mathrm{U}$ *versus* $\langle \mathrm{v}^2\rangle/(2\mathrm{U}^2)$. *The circles are all the measurements and the straight line, ordinate = 1·036 × abscissa − 0·0003, is a least squares fit to the data where the bias is less than 5%*

and the unit vector in the direction of the instantaneous horizontal component of the wind velocity

$$\mathbf{t}_l = \frac{\tilde{u}_l}{\tilde{s}_l}\mathbf{i} + \frac{\tilde{v}_l}{\tilde{s}_l}\mathbf{j} \tag{23}$$

where the horizontal unit vectors **i** and **j** define the intrinsic Cartesian co-ordinate system of the sonic. There are $N = 6000$ instantaneous measurements in each 10 min period and for each of these we calculate the magnitude of the mean wind vector

$$U_{\text{sonic}} = \sqrt{\left(\frac{1}{N}\sum_{l=1}^{N}\tilde{u}_l\right)^2 + \left(\frac{1}{N}\sum_{l=1}^{N}\tilde{u}_l\right)^2} \tag{24}$$

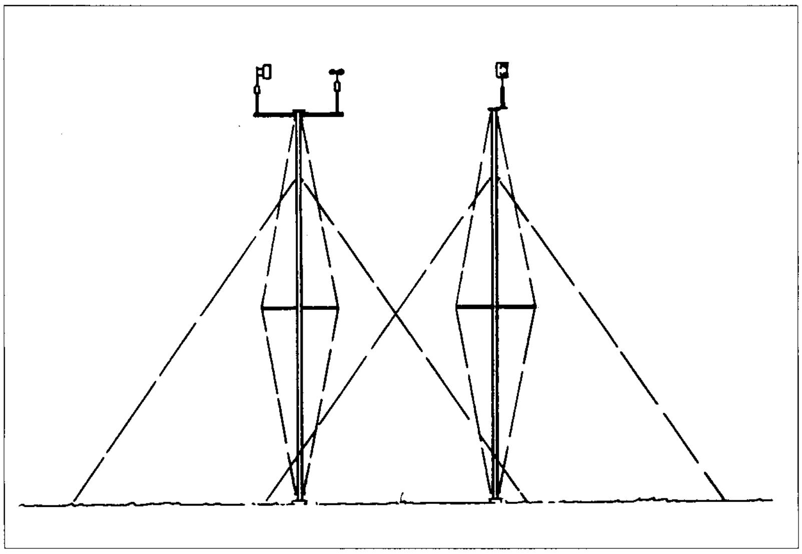

Figure 11. Instrumentation at Borris experiment. The two masts are 6 m high and separated by 3 m in the geographical direction 5°. The northernmost mast has a 1·2 m boom in the same direction. On this boom a cup anemometer of the type shown in Figure 1 is mounted at the south end and a wind vane at the other. The height of the cup rotor is about 6·6 m. The sonic, a Solent Ultrasonic Anemometer model 1012R, is mounted on the top of the southernmost mast such the centre is also at a height of about 6·6 m

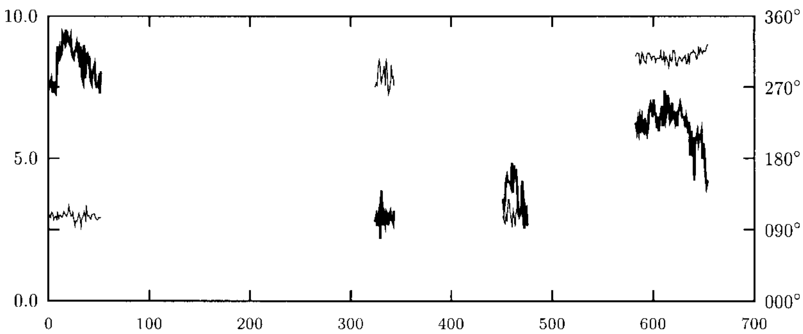

Figure 12. Time series of 10 min averages of direction (thin lines) and wind speed (thick lines) based on measurements by wind vane and cup anemometer respectively. Wind speed is measured in m s^{-1} and time in units of 10 min. In all four cases the angle between the wind direction and the line between the masts is larger than about 60°

and the sonic mean speed

$$\langle|\mathcal{U}|_{\rm h}\rangle_{\rm sonic} = \frac{1}{N}\sum_{l=1}^{N}\tilde{s}_l \tag{25}$$

This last quantity should be equal to $\langle|\mathcal{U}_{\rm h}|\rangle$ measured by the cup anemometer. The left frame of Figure 13 shows $\langle|\mathcal{U}|_{\rm h}\rangle_{\rm sonic}$ plotted versus $\langle|\mathcal{U}|_{\rm h}\rangle$ for all the measurements. A linear fit results in

$$\langle|\mathcal{U}|_{\rm h}\rangle_{\rm sonic} = a \times \langle|\mathcal{U}|_{\rm h}\rangle + b \tag{26}$$

where $a = 1{\cdot}0409 \pm 0{\cdot}0025$ and $b = -0{\cdot}207 \pm 0{\cdot}016$. We see that when $\langle|\mathcal{U}|_{\rm h}\rangle \geqslant 5\,{\rm m\,s^{-1}}$, the emulated sonic speed is larger than the wind speed measured by the cup anemometer. The right frame of Figure 13 shows that when $\langle|\mathcal{U}|_{\rm h}\rangle \gtrsim 7\,{\rm m\,s^{-1}}$, even the sonic mean vector $U_{\rm sonic}$ is larger than $\langle|\mathcal{U}|_{\rm h}\rangle$. This can

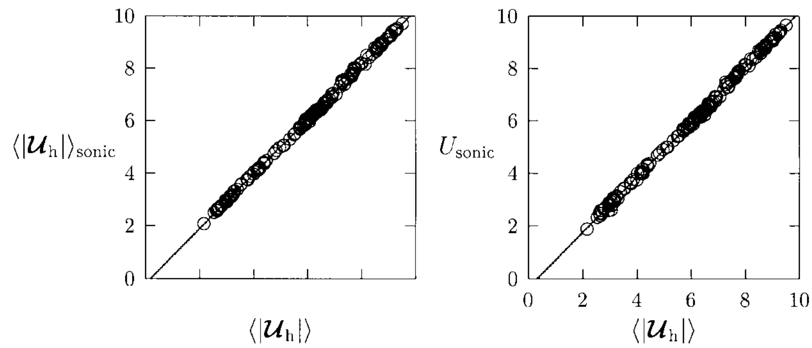

Figure 13. Left frame: sonic emulated mean speed versus $\langle|\mathcal{U}|_h\rangle$. Right frame: sonic mean vector velocity versus $\langle|\mathcal{U}|_h\rangle$

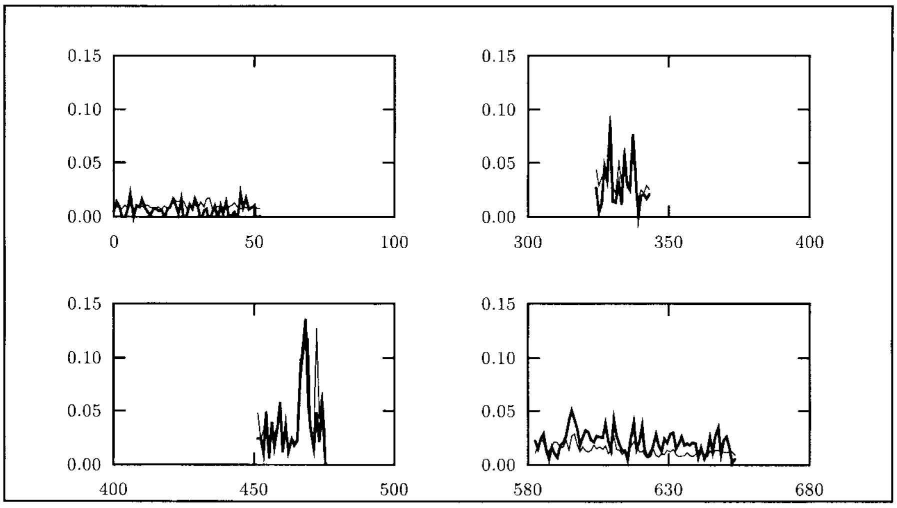

Figure 14. Relative bias $(U - U_{sonic})/U_{sonic}$ (thick line) and $\langle v^2\rangle_{sonic}/(2U^2_{sonic})$ (thin line) as a function of time, corresponding to four periods shown in Figure 12

certainly not be attributed to cup anemometer overspeeding. Rather it should be interpreted as a lack of reliable intercalibration between the sonic and the cup. In fact, we used the sonic calibration provided by the manufacturer and an independent, careful cup calibration of our own. Consequently, we assume that we may safely use (26) to recalibrate the sonic to be consistent with the cup.

The magnitude of the mean wind vector and the lateral variance were now recalculated by means of (24) and

$$\langle v^2\rangle_{\text{sonic}} = U^2_{\text{sonic}}\left[1 - \left(\frac{1}{N}\sum_{l=1}^{N}\frac{\tilde{u}_l}{\tilde{s}_l}\right)^2 - \left(\frac{1}{N}\sum_{l=1}^{N}\frac{\tilde{v}_l}{\tilde{s}_l}\right)^2\right] \tag{27}$$

Figure 14 shows four frames of the cup mean speed U minus the magnitude of the mean wind vector U_{sonic}, normalized by U_{sonic}, as a function of time together with the prediction $\delta_v = \langle v^2\rangle_{\text{sonic}}/(2U^2_{\text{sonic}})$ according to (13).

The relative bias seems to be well accounted for by (13) when it is large. In particular, in the upper right frame and the lower left frame the two curves track quite well. In the lower right frame, where the systematic error lies between 0% and 5%, δ_v appears to be about half that. For this particular set of data we made a special linear fit between the sonic emulated mean speed $\langle|\mathcal{U}|_h\rangle_{\text{sonic}}$ and the cup mean speed

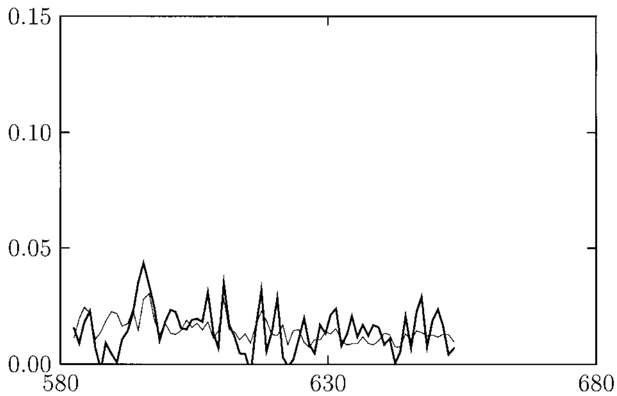

Figure 15. Fourth frame of Figure 14 after second recalibration

$\langle|\mathcal{U}|_h\rangle$. This modified the fourth frame of Figure 14 in such a way that the prediction (13) became rather good, as Figure 15 shows. Admittedly, this second recalibration weakens the test.

Conclusions

Let us summarize the advantages of using a cup anemometer.

1. The instrument is robust.
2. It has a vertical symmetry axis and is consequently equally sensitive to winds from all directions.
3. The calibration is very linear.
4. The measuring accuracy is high. The mean wind can be determined within about 1%.
5. The distance constant l_0 can for special purposes be made so small—at the expense of ruggedness, of course—that it can compete with the sonic anemometer with respect to temporal resolution.

The high degree of accuracy does not come for free. It is necessary to be careful with the construction such that the angular response $g(\vartheta)$ is close to the cosine response. This reduces the δ_w error from the vertical fluctuations of the wind velocity. It is also advantageous to find a good compromise between ruggedness and fast reaction to changes in the wind velocity; for example, light materials such as styrofoam might lead to a small distance constant, but the strength of the instrument is low. A distance constant of 1–2 m is suitable for most routine measurements where it is a matter of determining the mean wind speed with a δ_u bias less than about 1%. The most serious error stems from the wind direction fluctuations. This systematic error, however, can be eliminated, as has been shown, by combining the cup anemometer with a wind vane.

Is the accuracy of 1% sufficient to satisfy the user?

We have mentioned that wind turbine manufacturers and customers are rather demanding, but it seems that the cup anemometer should be all right for their applications.

Another case where it is important to obtain high accuracy is the study of the lowest 100 m of the atmosphere in connection with the determination of fluxes of momentum and scalar quantities to and from the lower surface. Here it is a matter of measuring accurately the wind speed change with height. Typically a 20 m mast is erected and the wind profile measured by means of three or four anemometers. In certain situations the change from 10 to 20 m is as small as 5% and one must therefore have an accuracy of about 1%, the same demand as that of the wind power people. As pointed out, this accuracy can be obtained, but one must be careful with the mounting of the anemometers. We have to make sure that the flow field is not disturbed by booms, the mast and other instruments. There are actually situations where disturbance is unavoidable; for example, when the wind blows through the mast. In these situations the measured wind profiles become unrealistic, but of course this is a more general problem which is not connected to what kind of anemometer is used.

If the cup anemometer is discredited because of overspeeding, it seems that is not justified. It is a very accurate and reliable instrument and probably much better than its reputation.

Acknowledgements

To a very large extent the text is a translation from a Danish article in the journal *Vejret* (*The Weather*).[17] The measurements at the Risø mast were carried out in co-operation with Arent Hansen and Mike Courtney of Risø. Arent Hansen designed the combined cup–vane anemometer and Mike Courtney programmed the data logging. The measurements in Borris were part of a larger diffusion experiment with Torben Mikkelsen as PI. Hans E. Jørgensen extracted the data we have used here. Figures 1, 7, 8 and 11 were produced by Arent Hansen. Jakob Mann offered valuable comments.

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